Integrated Production-Distribution Planning
An optimization model for mixed-model assembly lines

Felix Zesch

Bernd Hellingrath

*) 4flow AG, 10587, Berlin, Germany
E-mail: f.zesch@4flow.de, Tel: +49-30-379400

**) Münster School of Business and Economics, Chair of Information Systems and Logistics,
48149, Münster, Germany
E-mail: hellingrath@wi.uni-muenster.de, Tel: +49-251-83-38000;
ABSTRACT

Purpose of this paper

This paper presents an optimization approach for the integrated planning of production and logistics for vehicle manufacturers in a multi-site production network with multi-modal two-stage distribution. The novelty lies in the integration of production sequencing constraints stemming from mixed-model assembly lines and the effort for high transport vehicle utilization as a contribution to a greener supply chain.

Design/methodology/approach

Using industry data from a German vehicle manufacturer, customer orders in a multi-modal two-stage distribution network are allocated to daily buckets of several plants explicitly considering $H_o:N_o$ sequencing constraints from multi-model assembly lines. We use OptimJ to formulate the model and solve it subsequently with CPLEX. Due to the np-hardness of the problem, a solution to optimality is generally not feasible.

Findings

The calculated utilization of transports is generally over 90 % for trucks and trains, indicating a successful load optimization.

For a real-world problem we obtain after eight hours a solution with a MIP gap of 5.15 %. It could thus be worthwhile to test heuristics and other approaches such as valid cuts to improve the solution quality and speed.

Research limitations/implications

The approach measures the transportation vehicles’ travel time in whole periods. It does not consider continuous planning nor routing decisions. Heuristics should be tested with this problem as an alternative solution technique. Simulation would provide a more detailed and reliable evaluation of the effects.

Practical implications

The model presented can serve as a basis for a logistics assistance system supporting the planning of production and outbound distribution of cars.

What is original/value of paper

The integration of production and logistics for manufacturers producing on mixed-model assembly lines has not yet been explored. Our model shows a way to combine sequencing constraints and a two-stage distribution network for the assignment of orders to daily buckets in a production network of vehicle manufacturers.

Keywords: Integrated production distribution planning, automotive industry, binary integer model, logistics modeling, multi-modal planning, multi-site production, two-stage distribution
INTEGRATED PRODUCTION-DISTRIBUTION PLANNING – AN OPTIMIZATION MODEL FOR MIXED-MODEL ASSEMBLY LINES

1.1. Production and distribution in the automotive industry

Today’s automotive industry is characterized by mass customization with high volumes and a large variety of different options, yielding several billions of possible models (Meyr, 2004). Production takes place on mixed-model assembly lines which are characterized by a substantial reduction in setup times and cost due to the application of flexible workers and machinery, so that different products can be jointly manufactured in intermixed product sequences (lot size of one) on the same line. (Boysen, Fliedner, & Scholl, 2009). Cost are driven through the production sequence which is subject to workload constraints at the stations linked to the car’s different options (e.g. electric sunroof yes/no).

Distribution of finished vehicles in the automotive industry is mainly done by road and rail as well as deep sea vessels for ocean transport, each of these transportation modes having a different capacity. A two-stage distribution network with transport from plant to distribution-centers and then to dealerships is common (Nozick & Turnquist, 2001). The integration of production and distribution is supported through the large amount of capital bound in finished vehicles waiting for their delivery. Another major factor are ecological concerns demanding for high utilization and a shift from road to rail transport.

1.2. Integration of production and distribution

Technically, the integrated production and distribution planning strives to find a solution that is better than the result of two separate optimizations in production and distribution. The general financial advantages of this approach have already been shown on many different planning levels in many sectors (Erengüç, Simpson, & Vakharia, 1999), (Chen, 2004), (Chen, 2008). For the automotive industry, they have yet to be demonstrated, but there are ongoing efforts in this direction. In recent approaches, the focus has been on integrated production-distribution models on the execution level, esp. in sequencing, and leading vehicle manufacturers pursue efforts in this field, cf. e.g. (Decker, 2009) and (Scholz-Reiter, Böse, Dizdar, & Windt, 2008).

The environment-friendlier rail and waterway transports need a longer planning lead time and more volume in order to be competitive with road transportation. The production planning of a vehicle manufacturer can already consider these requirements on the tactical level with a horizon of several weeks to one year. This would yield a higher and more efficient usage of rail and water transports. Timely planned transports on these carriers could help decrease transport costs and lead time.

Another incentive for more transports using rail and waterway is the increased regulation of road transport, which is partially due to the increased societal awareness of climate change and carbon dioxide as one of its major triggers (IPCC, 2007, BMW, 2008).

There is thus a scientific, an economic and a political motivation for the closer investigation of integrated production-distribution planning in the automotive industry on a tactical level.
1.3. Methodology

As a basis for the optimization model we use formerly developed requirements for integrated production-distribution planning from the automotive industry (Zesch & Hellingrath, 2009). Special consideration is given to the inclusion of sequencing constraints on the precedent planning level. We design an optimization model to evaluate the possible advantages of integrated planning. The model is formulated using OptimJ (Ateji) and then solved with CPLEX (IBM ILOG).

We evaluate different use cases to discover situations where the benefits stemming from an integrated planning are greatest.

2. Literature Review

The integrated planning of production and distribution is especially important for perishable goods, but can also lead to significant potentials for cost cutting in other industries (Chen, 2004). The integration can occur on the strategic, tactical or operative level, each having a different set of requirements and goals (Erengüç et al., 1999, Chen, 2004, Chen, 2008).

The planning of the production program or master schedule occurs usually on a horizon of several weeks to one year (Jodlbauer, 2008, pp. 110f). Main decisions concern the amount of products to be produced and transported in one period. On the precedent strategic level, decisions about plant and hub location and network design are made. On the operative level the detailed assignment of orders to resources such as machines and transports as well as routing are dealt with (Chen, 2004, p. 714).

The body of literature on integrated production-distribution planning is rich with a recent focus on sequencing decisions (Chen, 2008 gives an overview). Only seldom is there a specific link to the automotive industry and its particularities such as mixed-model assembly lines. The only paper dealing with the integrated production and distribution in the automotive industry on a tactical level that we found is (Torabi & Hassini, 2008). Yet they assume set-up costs for the automotive industry though a characteristic of mixed-model assembly used in the automotive industry is that set-up costs are negligible (Boysen et al., 2009) (but in the paint shop before production). Instead, the infringement of sequencing rules usually leads to additional costs (Boysen, Fliedner, & Scholl, 2007).

At least two models for production planning consider sequencing constraints on the precedent planning level (Boysen, 2005, Volling, 2009) but none of these includes distribution planning decisions. From the table 5.1 in the appendix summarizing the literature can be seen that contributions dealing with integrated production distribution planning neglect sequencing constraints and those dealing with production planning and sequencing constraint have not yet included distribution planning.

Three basic approaches can be identified for order sequencing on mixed-model assembly lines: Mixed-model sequencing, car sequencing and level scheduling (Boysen et al., 2009). In this paper we focus on the car sequencing approach, which strives to avoid the significant effort of data collection that accompanies mixed-model sequencing. Car sequencing attempts to minimize sequence-dependent work overload in an implicit manner. This is achieved by formulating a set of sequencing rules of type \( H_o ; N_o \), which postulate that among \( N_o \) subsequent sequence positions at most \( H_o \) occurrences of a certain option \( o \) are allowed. If a sequence is found which does not violate such rules, work overload can be avoided. Even if
avoidance is not fully possible, the work overload is supposed to be the lower the fewer rules are violated.

3. Optimization Model

3.1. Variables

The following tables 3.1 and 3.2 present the variables and their meaning.

**Table 3.1 Decision variables**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{itl}$</td>
<td>Binary variable, assigning order $i$ to bucket $t$ on line $l$</td>
</tr>
<tr>
<td>$s_{itl}$</td>
<td>Number of orders, which go from $w$ to $d$ in $t$ per $r$</td>
</tr>
<tr>
<td>$s_{trwd}$</td>
<td>Number of orders, which go from $d$ to $p$ in $t$ per $r$</td>
</tr>
<tr>
<td>$q_{trwd}$</td>
<td>Number of vehicles of type $r$ driving in period $t$ from $w$ to $d$</td>
</tr>
<tr>
<td>$q_{trdp}$</td>
<td>Number of vehicles of type $r$ driving in period $t$ from $d$ to $p$</td>
</tr>
</tbody>
</table>

**Table 3.2 Parameters**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{it}$</td>
<td>Pseudo-cost for the arrival of order $i$ in period $t$</td>
</tr>
<tr>
<td>$b_{io}$</td>
<td>Binary variable indicating whether order $i$ has option $o$.</td>
</tr>
<tr>
<td>$B_{ot}$</td>
<td>Number of available options $o$ in period $t$</td>
</tr>
<tr>
<td>$\beta_{ot}$</td>
<td>Damping coefficient for $Ho$-$No$-rules</td>
</tr>
<tr>
<td>$c_{rwd}$</td>
<td>Transport costs for means of transportation $r$ on the arc from $w$ to $d$</td>
</tr>
<tr>
<td>$c_{rtd}$</td>
<td>Transport costs for means of transportation $r$ on the arc from $d$ to $p$</td>
</tr>
<tr>
<td>$C$</td>
<td>Inventory carrying costs per period.</td>
</tr>
<tr>
<td>$e_{il}$</td>
<td>Binary variable indicating whether order $i$ can be built on line $l$.</td>
</tr>
<tr>
<td>$g_{tr}$</td>
<td>Speed of means of transportation $r$ expressed as travelling time in periods.</td>
</tr>
<tr>
<td>$I$</td>
<td>Number of orders</td>
</tr>
<tr>
<td>$K_{r}$</td>
<td>Capacity of means of transportation $r$</td>
</tr>
<tr>
<td>$\lambda_{wi}$</td>
<td>Binary variable assigning a line to a plant</td>
</tr>
<tr>
<td>$m_{it}$</td>
<td>Production costs of order $i$ in period $t$</td>
</tr>
<tr>
<td>$N_{it}^{\text{max}}$</td>
<td>Maximum capacity of line $i$ in period $t$</td>
</tr>
<tr>
<td>$N_{it}^{\text{min}}$</td>
<td>Minimum utilization of line $i$ in period $t$</td>
</tr>
<tr>
<td>$p_{d}$</td>
<td>Collection time at distribution center $d$</td>
</tr>
<tr>
<td>$q_{d}^{\text{max}}$</td>
<td>Maximum capacity at distribution center $d$ in period $t$</td>
</tr>
<tr>
<td>$q_{d}^{\text{min}}$</td>
<td>Target minimum stocks at distribution center $d$ in period $t$</td>
</tr>
<tr>
<td>$r_{it}$</td>
<td>Iterative variable</td>
</tr>
<tr>
<td>$T$</td>
<td>Total number of periods considered</td>
</tr>
<tr>
<td>$u_{it}$</td>
<td>Binary variable, which is 1, if plant $w$ delivers to distribution center $d$ per $r$</td>
</tr>
<tr>
<td>$v_{rwp}$</td>
<td>Binary variable, which is 1, if customer $p$ is served from distribution center $d$ per $r$</td>
</tr>
</tbody>
</table>
3.2. **Objective function**

We consider six objectives in our problem:

1. Production cost PK
2. Deviation cost AK
3. Transport cost on first stage TK\(_1\)
4. Transport cost on second stage TK\(_2\)
5. Inventory carrying cost at plants LK\(_1\)
6. Inventory carrying cost at distribution centers LK\(_2\)

The first objective is the production cost, that is common to most production-distribution problems. The second objective AK is related to the production on mixed-model assembly lines and has been treated recently by (Boysen, 2005) and (Volling, 2009) for the automotive industry. The four remaining objectives are important as well (cf. e.g. Holweg & Miemczyk, 2003), but have not yet been treated in the context of a vehicle manufacturer's production planning.

*Formula 1: Objective function*

$$\min_{x_{it}, d_{it}, x_{itl}, d_{itl}} \, PK + AK + TK_1 + TK_2 + LK_1 + LK_2$$

### 3.2.1. Production cost

Production cost comprises the time-independent cost of manufacturing including costs for material and work force. They are model- and line-dependent.

*Formula 2: Production cost in the objective function*

$$PK = \sum_{v} \sum_{i} \sum_{l} m_{il} x_{il}$$

### 3.2.2. Deviation cost

Deviation cost is calculative cost for the deviation between the date desired by the customer and the actual delivery date. They are time-dependent and given through \(a_{it}\) where \(i\) is the order and \(t\) the period. The deviation cost is described as a bathtub-shaped function, punishing late and early delivery while promoting on-time delivery, yet not necessarily preferring one single period (but possibly a set of good delivery periods).

*Formula 3: Deviation cost in the objective function*

$$AK = \sum_{v} \sum_{i} \sum_{l} a_{it} x_{itl}$$
Since these costs can be changed for every single order, a bespoke treatment of orders depending on their characteristics (e.g. longer possible delivery horizon for fleet vehicles, or shorter time window for private customer orders) is possible.

These costs are computed before the optimization thus reducing complexity. They consider already the vehicle’s travelling time. As a rough approximation of reality and in order to reduce complexity, the travelling time measured in whole periods is supposed to be equal from each plant to each dealer. This assumption holds only for plants located sufficiently close one to another.

### 3.2.3. Transport cost on two stages

The amount of transport vehicles needed per period and the cost per transport sum up to the transport cost for the first stage of the distribution network from plants to distribution centers. For ocean transport, the first stage consists of the transport from the plant to the port and the second from port to port. It is not meaningful to consider further distribution decisions after the port due to the low schedule reliability of deep-sea vessels (Vernimmen, Dullaert, & Engelen, 2007) and the large amount of cars at disposal for further delivery.

**Formula 4: Transport cost of the first stage in the objective function**

\[
TK_1 = \sum_{vl} \sum_{vr} \sum_{vd} \sum_{vp} q_{trwd} C_{rwd}
\]

The use of arc-dependent transportation cost may result in orders being produced earlier for a more expensive transport, since an empty space on an expensive transport causes higher cost than an empty space on an inexpensive transport.

Several transport modes are possible for the same arc. They are described through the index \( r \). These modes are characterized by different capacities and costs. Travel times measured in whole periods are supposed to be equal for all modes.

The transport cost on the second stage is formulated similarly to those on the first stage.

**Formula 5: Transport cost of the first stage in the objective function**

\[
TK_2 = \sum_{vl} \sum_{vr} \sum_{vd} \sum_{vp} q_{trdp} C_{rdp}
\]

### 3.2.4. Inventory carrying cost on two stages

After production, an order \( i \) causes inventory carrying cost. These costs occur at the plant and at the transshipment point in the distribution center. Inventory carrying cost during transports is included through the consideration of different speeds for different transport modes.

The inventory carrying cost is calculated on the basis of the difference between the incoming and the outgoing flow. Due to the granularity chosen, only inventory carrying cost due to stocking for more than one period are considered.

**Formula 6: Inventory carrying cost on the first stage in the objective function**

\[
LK_1 = \sum_{\tau=1}^{T} \sum_{vl} \sum_{i=1}^{l} \left( \sum_{vl} \sum_{vI} x_{il} \lambda_{trwd} - \sum_{vd} \sum_{vr} s_{trwd} \right) \cdot C
\]
The parameter for inventory carrying cost of one period \( C \) is equal over all models.
The inventory carrying cost on the stage of the distribution centers is formulated similarly:

**Formula 7: Inventory carrying cost on the second stage in the objective function**

\[
LK_2 = \sum_{\tau=1}^{T} \sum_{\forall d} \left( \sum_{\forall r} \sum_{t=1}^{\tau} \sum_{\forall w} s_{twd} - \sum_{\forall r} \sum_{t=1}^{\tau} \sum_{\forall p} s_{trdp} \right) \cdot C
\]

### 3.3. Constraints

#### 3.3.1. Distribution

All orders need to reach their final destination by a valid means of transportation \( r \) until period \( T \) has elapsed. For transports on the first stage from plants to distribution centers this yields:

**Formula 8: All orders pass through a distribution center**

\[
\sum_{\forall t} \sum_{\forall w} \sum_{\forall t} x_{itw} = \sum_{\forall t} \sum_{\forall w} \sum_{\forall r} s_{twd} u_{rwd}, \quad \forall W
\]

And for the second stage from distribution centers \( d \) to dealers \( p \):

**Formula 9: All orders are delivered to a dealer**

\[
\sum_{\forall r} \sum_{\forall w} \sum_{\forall t} s_{twd} u_{rwd} = \sum_{\forall r} \sum_{\forall p} \sum_{\forall t} s_{trdp} v_{rwp}, \quad \forall d
\]

Orders to be reloaded after arrival at the plant’s yard or at the distribution center need to have arrived already under consideration of a transport mode’s \( r \) travel time \( g_r \).

**Formula 10: Orders to be loaded need to be already produced**

\[
\sum_{t=1}^{\tau} \left( \sum_{\forall t} \sum_{\forall w} x_{itw} \lambda_{itw} - \sum_{\forall r} \sum_{t=1}^{\tau} s_{twd} u_{rwd} \right) \geq 0, \quad \forall W, \tau = 1..T
\]

**Formula 11: Orders can only be reloaded after arrival**

\[
\sum_{\forall t=1}^{\tau-g_r} \sum_{\forall w} \sum_{\forall t} s_{twd} u_{rwd} - \sum_{\forall r} \sum_{t=1}^{\tau} s_{trdp} v_{rwp} \geq 0, \quad \forall d, \tau = 1..T
\]
The next constraint assures that all orders $i$ arrive at their destination $p$.

**Formula 12:** Each dealer receives the desired amount of orders and the right orders:

$$\sum_{\forall i} z_{ip} = \sum_{\forall i} \sum_{\forall r} \sum_{\forall d} s_{rdp}, \quad \forall p$$

Formula 13 limits the solution space of $x_{itl}$ to those for which a viable path from a plant to a dealer exists. On the right-hand side order-assignment $x_{itl}$ is linked to the routing variables $u_{rwd}$ and $v_{rdp}$. By limiting the scope of this constraint to all $z_{ip}=1$ the number of constraints is reduced significantly.

**Formula 13:** All orders are produced for their destinations

$$z_{ip} \leq \sum_{\forall i} \sum_{\forall l} \sum_{\forall w} \sum_{\forall d} x_{ilw} \lambda_{lw} \cdot \sum_{\forall r} u_{rwd} \cdot \sum_{\forall r} v_{rdp}, \quad \forall i, p : z_{ip} = 1$$

Formula 13 assures only the production of all orders, but not their delivery. The delivery is assured through the flow constraints (8), (9), (10) and (11).

**Formula 14:** The capacity of a distribution center is always respected.

$$\sum_{\forall r} \sum_{t=1}^{\tau} s_{rwd} u_{rwd} - \sum_{\forall r} \sum_{t=1}^{\tau} \sum_{\forall r} s_{rdp} v_{rdp} \leq q_{d\tau}^{\text{max}}, \quad \forall d, \tau = 1..T$$

A high utilization of distribution transports is achieved through the integer number of vehicles needed $q$ (cf. formulas 26 and 27) and the following constraints, where the number of units transported $s$ divided by the transport vehicle capacity $k_r$ serves as a lower bound:

**Formula 15:** Lower bound for the number of transport vehicles first stage

$$q_{rwd} \geq \frac{s_{rwd}}{k_r}, \quad \forall t, r, w, d : u_{rwd} \neq 0$$
3.3.2. Production and inbound logistics

The following necessary but not sufficient condition for the respect of $H_o,N_o$-sequencing constraints in all periods can be adjusted by the damping coefficient $\beta_o$ to distinguish between hard and soft constraints. Hard constraints are imposed through the assembly line layout and cannot be altered through short term measures like additional work force. Soft constrains can be altered through the employment of additional work force and may therefore more easily be disobeyed.

**Formula 17: Consideration of sequencing constraints**

$$\sum_{\forall i} b_{oi} x_{itl} \leq \beta_{oi} \frac{H_{oit}}{N_{oit}} \cdot N_{itl}^{max}, \quad \forall o,t,l,$$

Capacity and utilization are controlled through the following constraints. The number of options $o$ used per period can be leveled through

**Formula 18: Respect of component availability**

$$\sum_{\forall i} \sum_{\forall o} b_{oi} x_{itl} \leq B_{oi}, \quad \forall o,t$$

**Formula 19: Minimum utilization of each line is respected**

$$\sum_{\forall i} x_{itl} \geq N_{itl}^{min} d_{itl}, \quad \forall t,l$$

**Formula 20: Maximum utilization of each line is respected**

$$\sum_{\forall i} x_{itl} \leq N_{itl}^{max} d_{itl}, \quad \forall t,l$$

Since nowadays car models can only be built on accordingly planned assembly lines and not on any given one, the manufacturing feasibility has to be checked for each order and line:
Formula 21: The manufacturing feasibility is respected
\[ x_{il} \cdot e_{il} = x_{il}, \quad \forall i, l \]

Formula 22: Each order can be assigned only once
\[ \sum_{i_l} \sum_{i} x_{il} = 1, \quad \forall i \]

### 3.3.3. Number range constraints

Formula 23: Binary constraint for the order assignment
\[ x_{il} \in \mathbb{N}, \quad x_{il} \in \{0, d_{il} \cdot e_{il}\} \]

Formula 24: Integer constraint for transport demand first stage
\[ s_{rwd} \in \mathbb{N}, \quad s_{rwd} \begin{cases} = 0, u_{rwd} = 0 \\ \geq 0, u_{rwd} = 1 \end{cases} \]

Formula 25: Integer constraint for transport demand second stage
\[ s_{rdp} \in \mathbb{N}, \quad s_{rdp} \begin{cases} = 0, v_{rdp} = 0 \\ \geq 0, v_{rdp} = 1 \end{cases} \]

Formula 26: Integer constraint for the number of transport vehicles first stage
\[ q_{rwd} \in \mathbb{N}, \quad q_{rwd} \begin{cases} = 0, u_{rwd} = 0 \\ \geq 0, u_{rwd} = 1 \end{cases} \]

Formula 27: Integer constraint for the number of transport vehicles second stage
\[ q_{rdp} \in \mathbb{N}, \quad q_{rdp} \begin{cases} = 0, v_{rdp} = 0 \\ \geq 0, v_{rdp} = 1 \end{cases} \]

### 3.4. Assumptions

#### 3.4.1. Structure
Each order has only one dealer as destination
\[ \sum_{\forall p} z_{ip} = 1, \quad \forall i \]
Each distribution center is supplied by at least one plant \( \sum_{w} \sum_{r} u_{wd} \geq 1, \forall d \)

Each dealer is supplied by exactly one distribution center (an argumentation for this assumption is given by Benjaafar, ElHafsi, & de Véricourt, 2004 and Chan & Simchi-Levi, 1998): \( \sum_{v} \sum_{r} v_{rdp} = 1, \forall p \)

### 3.5. Example scenario

To put the model to a test we consider a scenario inspired from a real-world example of a German vehicle manufacturer and modified with the following main characteristics:

#### Table 3.3 Example scenario: Structure

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plants</td>
<td>3</td>
</tr>
<tr>
<td>Production lines</td>
<td>7</td>
</tr>
<tr>
<td>DC</td>
<td>17</td>
</tr>
<tr>
<td>Dealers</td>
<td>100</td>
</tr>
<tr>
<td>Options</td>
<td>41</td>
</tr>
<tr>
<td>Modes of transport</td>
<td>3</td>
</tr>
<tr>
<td>Orders</td>
<td>10,000</td>
</tr>
<tr>
<td>Periods</td>
<td>20</td>
</tr>
<tr>
<td>Options</td>
<td>41</td>
</tr>
</tbody>
</table>

#### Table 3.4 Example scenario: Distribution cost for different modes

<table>
<thead>
<tr>
<th>Transportation vehicle</th>
<th>Mode</th>
<th>Max capacity (vehicles)</th>
<th>Fix cost per trip</th>
<th>Variable cost per km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truck</td>
<td>Road</td>
<td>8</td>
<td>0,00 €</td>
<td>1,15 €</td>
</tr>
<tr>
<td>Waggon, whole train</td>
<td>Rail</td>
<td>10</td>
<td>0,00 €</td>
<td>0,15 €</td>
</tr>
<tr>
<td>Single wagon</td>
<td>Rail</td>
<td>10</td>
<td>600,00 €</td>
<td>0,85 €</td>
</tr>
<tr>
<td>Engine</td>
<td>Rail</td>
<td>650</td>
<td>2,400 €</td>
<td>7,60 €</td>
</tr>
<tr>
<td>Deep sea vessel</td>
<td>Water</td>
<td>3000</td>
<td></td>
<td>230,00 €</td>
</tr>
</tbody>
</table>

In our model, we distinguish between truck, single wagon and full train for land-based transports and consider the deep sea vessel for maritime transports. The figures have been taken from industry and are slightly altered.
4. Results and Discussion

4.1. Model size and solvability

The given scenario yields a model with 132,760 constraints. 92 % of the constraints are contributed by the three formulas 16 (77 %), 21 (8 %) and 12 (8%).

To reduce the number of constraints from Formula 15 and Formula 16, a ceiling function would be sensible. Instead of the constraints the ceiling function could be inserted in the objective function as \[
\frac{S_{\text{trwd}}}{K_r}
\]
such that \(q_{\text{trwd}}\) and \(q_{\text{trdp}}\) would no longer be needed.

Unfortunately, the ceiling function is not linear and the problem becomes much harder to solve this way. The taken approach of a increase of the model size seems to be less cumbersome. An uncommon trait of the optimization is that the decision variables \(q_{\text{trdw}}\) and \(q_{\text{trdp}}\) are not part of the objective function.

Obviously, abstracting from the real supply chain of a distribution network by grouping several dealers to one is a viable option to increase the performance. This may even be useful to represent outbound distribution delivery runs.

The resulting binary integer problem is np-hard (Gök en & Erel, 1998) and thus becomes computationally intractable for large problems.

The number of decision variables for this problem is 1,610,120 with \(x_{il}\) accounting for 87 %.

The solution space can be limited through the constraints in section 3.3.3, bounding the range in which the decision variables can be altered and thus facilitating the solution of the problem.

4.2. Results

Due to the combinatorial nature of the problem, an optimal solution is very hard to find. The optimization run for the mentioned real-world problem yielded a duality gap of 5.15 %. 5 % of the production and distribution cost being still a large amount of money, further investigation for apt solution methods should be undertaken.

The capacity utilization of the transportation vehicles is always over 90 %. The high utilization attained can contribute to the ambition to shift more goods from road to rail transport. Their use depends heavily on the underlying cost function, yet.

4.3. Computation time

On a system with 8 Intel Quad Core CPUs at 2.3 GHz, 256 GB of RAM and a 64-bit java system, the solution for the case mentioned was reached after 8 hours. This shows that in principal the model is applicable to real-world problems, since such a computation run would not be made daily in a production system but rather weekly.

4.4. Limitations of the model

The approach measures the transportation vehicles’ travel time in whole periods. It does not consider continuous planning nor routing decisions. Delivery runs to several dealers are not explicitly allowed for, but could be represented through the grouping of dealers lying on one delivery run to one meta-dealer. Rolling horizon planning is not implemented, such that the solution would be altered in every planning cycle. Consideration of these factors will be addressed by further research.
The approach measures the transportation vehicles’ travel time in entire planning periods. Long planning periods thus cause a coarse consideration of time but make the model easier to solve, whereas a fine-grained representation of travel time increases the number of planning periods and makes the model harder to solve.

5. Summary

In this article, we presented an optimization approach for the integrated planning of production and logistics for vehicle manufacturers in a multi-site production network with multi-modal two-stage distribution. The novelty lies in the integration of production sequencing constraints.

Using industry data from a German vehicle manufacturer, customer orders in a multi-modal two-stage distribution network are allocated to daily buckets of several plants explicitly considering $H_o:N_o$ sequencing constraints from multi-model assembly lines.

The problem is np-hard and thus cannot be solved to optimality for large instances. Through an intelligent formulation of constraints, the solution space can be bounded to increase the solution speed.

The model does not consider continuous planning nor routing decisions. Delivery runs can only be represented by aggregating the concerning dealers.

The model presented can serve as a basis for a logistics assistance system supporting the planning of production and outbound distribution of cars.

5.1. Opportunities for future research

5.1.1. Validation of the approach

Further research needs to be done to compare these results in different cases to a reference model based on the currently dominating approach of separately treating production and distribution.

For a realistic evaluation of the effects, a corresponding simulation model needs to be built.

5.1.2. More detailed production planning

The assignment to daily buckets in this model as part of the production planning is done according to (Boysen, 2005) through the respect of $H_o:N_o$-type sequencing constraints. Instead of considering $H_o:N_o$-type sequencing constraints, a more detailed view of the production including station overload could be taken. Furthermore, restrictions and optimization goals stemming from inbound logistics may be a useful add-on.

In practice, the production cost of vehicles differs from plant to plant. This can be incorporated by a line-specific cost factor $a_{itl}$ instead of $a_{it}$.

5.1.3. Application of known and new heuristics to the problem

Known heuristics for similar problems such as (Lejeune & Margot, 2008) should be tested. The development of bespoke new heuristics for this production-distribution problem is another interesting research path.
ACKNOWLEDGEMENTS

We would like to thank David Jorch, Dimitri Ratkovitch, Martin Markoff and Sebastian Finkemeyer of the university of Münster and the German Federal Ministry of Economy for their support of the InTerTrans-project.
## APPENDIX:

**Table 5.1 Overview over the literature**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fleet</td>
<td>Homogeneous</td>
<td>Homogeneous</td>
<td>Homogeneous</td>
<td>Heterogeneous</td>
<td>Heterogeneous</td>
<td>Heterogeneous</td>
<td>Heterogeneous</td>
<td>Homogeneous</td>
<td>Heterogeneous</td>
<td>Homogeneous</td>
<td>N/A</td>
<td>N/A</td>
<td>Homogeneous</td>
<td>Heterogeneous</td>
</tr>
<tr>
<td>#vehicles limited</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>N/A</td>
<td>Y</td>
<td>Y</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>add. Ext. LSP vehicles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix transportation cost</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>N/A</td>
<td>N/A</td>
<td>Heterogeneous</td>
<td></td>
</tr>
<tr>
<td>Variable transportation cost</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>N/A</td>
<td>N/A</td>
<td>Heterogeneous</td>
<td></td>
</tr>
<tr>
<td>Arc-specific cost</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>N/A</td>
<td>N/A</td>
<td>Heterogeneous</td>
<td></td>
</tr>
<tr>
<td>Setup Costs</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>N/A</td>
<td>N/A</td>
<td>Heterogeneous</td>
<td></td>
</tr>
<tr>
<td>Inventory Holding</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>N/A</td>
<td>N/A</td>
<td>Heterogeneous</td>
<td></td>
</tr>
<tr>
<td>Fix production cost</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variable production cost</td>
<td></td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>N/A</td>
<td>N/A</td>
<td>Heterogeneous</td>
<td></td>
</tr>
<tr>
<td>Usage of production capacity</td>
<td></td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>N/A</td>
<td>N/A</td>
<td>Heterogeneous</td>
<td></td>
</tr>
<tr>
<td>Backorder cost</td>
<td></td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Empties</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Travel time considered</td>
<td></td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>N/A</td>
<td>N/A</td>
<td>Heterogeneous</td>
<td></td>
</tr>
<tr>
<td>Routing considered</td>
<td></td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inbound stage</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Production sites</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distribution center</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Point of sale</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exchange rates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Customs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Products</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Backlogging</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sequencing criteria considered</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
REFERENCES

Ateji. OptimJ. http://www.ateji.com/optimj


